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TATIIBIA UTIVERSITY
OF SCIERCE AMD TECHTOLOGY

## FACULTY OF HEALTH, APPLIED SCIENCES AND NATURAL RESOURCES DEPARTMENT OF MATHEMATICS AND STATISTICS

| QUALIFICATION: Bachelor of Science in Applied Mathematics and Statistics |  |  |  |
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| QUALIFICATION CODE: | 07BSAM | LEVEL: | 5 |
| COURSE CODE: | LIA502S | COURSE CODE: | LINEAR ALGEBRA 1 |
| SESSION: | NOVEMBER 2022 | PAPER: | THEORY |
| DURATION: | 3 HOURS | MARKS: | 100 |


| FIRST OPPORTUNITY EXAMINATION QUESTION PAPER |  |
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| EXAMINER: | MR. GS MBOKOMA, DR. N CHERE |
| MODERATOR: | DR. DSI IIYAMBO |

## INSTRUCTIONS

1. Attempt all the questions in the booklet provided.
2. Show clearly all the steps used in the calculations.
3. All written work must be done in black or blue inked, and sketches must be done in pencil.

## PERMISSIBLE MATERIALS

1. Non-programmable calculator without a cover.

## Question 1

1.1 Let $\mathbf{u}=2 \mathbf{i}+2 \mathbf{j}+3 \mathbf{k}$ and $\mathbf{v}=-\mathbf{i}+\mathbf{j}+4 \mathbf{k}$.
a) Find the unit vector $\hat{\mathbf{u}}$ in the direction of $\mathbf{u}$.
b) Find the projection vector of $\mathbf{u}$ onto $\mathbf{v}$.
b) Find the angle (in degrees) between $\mathbf{u}$ and $\mathbf{v}$. Give you answer correct to 2 d.p. [7]
1.2 Determine the area of parallelogram whose adjacents sides are $\mathbf{a}=2 \mathbf{i}-4 \mathbf{j}+5 \mathbf{k}$ and $\mathbf{b}=$ $\mathbf{i}-2 \mathbf{j}-3 \mathbf{k}$. Leave your answer in surd form.
1.3 If $\mathbf{A}$ and $\mathbf{B}$ are vectors, then show that $(\mathbf{A}-\mathbf{B}) \times(\mathbf{A}+\mathbf{B})=2(\mathbf{A} \times \mathbf{B})$

## Question 2

2.1 Let A be a square matrix and let

$$
S=\frac{1}{2}\left(A+A^{T}\right) \quad \text { and } \quad P=\frac{1}{2}\left(A-A^{T}\right)
$$

a) Find $S+P$.
b) Show that $S$ is symmetric and $P$ is skew-symmetric.
c) If A is symmetric, then show that $S=A$ and $P=0$.
2.2 Consider the matrix $A=\left(\begin{array}{ccc}-1 & 1 & 2 \\ 3 & 0 & -5 \\ 1 & 7 & 2\end{array}\right)$.
a) Use the Cofactor expansion method along the second column to evaluate the determinant of $A$.
b) Is $A$ invertible? If it is, Use the Gauss-Jordan Elimination method to find $A^{-1}$. [14]
c) Find $\operatorname{det}\left(3(2 A)^{-1}\right)$.

## Question 3

Determine whether or not the vector $(-1,1,5)$ is a linear combination of the vectors $(1,2,3),(0,1,4)$ and $(2,3,6)$.

## Question 4

Let $W=\left\{(x, y, z) \in \mathbb{R}^{3} \mid 3 y+2 z=0\right\}$.
a) Verify that $W$ is a subspace of $\mathbb{R}^{3}$.
b) Find a basis for $W$.

